DOCUMENT RESUME

ED 127 138 SE 020 845

AUTHOR	Chrisman, Gerry L.; Wheatley, Grayson H.
TITLE	Formal Operational Thought and Learning Strategies in Mathematical Structures.
PUB DATE	Apr 76
NOTE	42p.; Paper presented at the annual meeting of the American Educational Research Association (San Francisco, California, April 19-23, 1976); Not available in hard copy due to light and broken type throughout
EDRS PRICE	MF-$0.83 Plus Postage. HC Not Available from EDRS.
DESCRIPTORS	*Cognitive Development; Instruction; *Learning; Learning Theories; *Mathematics Education; *Research; Secondary Education; Secondary School Mathematics; *Transfer of Training

ABSTRACT

Eighth-grade subjects, classified as formal or concrete operational, learned a mathematical group structure with either a pre-determined sequence of trials (Reception Mode) or a free choice of trials (Selection Mode). All subjects learned a similar group structure one week later in the Selection Mode. Formal subjects learned in fewer trials and used a more effective strategy on both tasks. Reception Mode subjects learned in fewer trials on Task 1. Reception training resulted in more efficient learning on the transfer task for concrete but not formal subjects. Subjects appeared to transfer rules but not imposed strategies. (Author/SD)

FORMAL OPERATIONAL THOUGHT AND LEARNING STRATEGIES IN MATHEMATICAL STRUCTURES

Gerry L. Chrisman
Department of Mathematics
Northern Illinois University
DeKalb, Illinois 60115

Grayson H. Wheatley
Department of Mathematics
Purdue University
West Lafayette, Indiana 47907

Paper Presented at:

1976 Annual Meeting, American Educational Research Association

San Francisco, California, April 19-23, 1976

Formal Operational Thought and Learning Strategies in Mathematical Structures

GERRY L. CHRISMAN, Northern Illinois University

GRAYSON H. WHEATLEY, Purdue University

Abstract

Eighth-grade subjects classified as formal or concrete operational learned a mathematical group structure with either a pre-determined sequence of trials (Reception Mode) or a free choice of trials (Selection Mode). All subjects learned a similar group structure one week later in the Selection Mode. Formal subjects learned in fewer trials and used a more effective strategy on both tasks. Reception Mode subjects learned in fewer trials on Task 1. Reception training resulted in more efficient learning on the transfer task for concrete but not formal subjects. Subjects appeared to transfer rules but not imposed strategies.

FORMAL OPERATIONAL THOUGHT AND LEARNING

STRATEGIES IN MATHEMATICAL STRUCTURES

Dienes and Jeeves (1965, 1970) identified a heirarchy of strategies used in learning certain mathematical group structures. Other researchers (Leskow and Smock, 1970; Tagatz, 1967; Yudin and Kates, 1963; Yudin, 1966) have shown a relationship between strategy use in various tasks and chronological age. Using group structure tasks, the present study investigated the dependence of strategy choice on stage of cognitive development.

Developmental Stages

Piaget and Inhelder (1969) have described four hypothesized stages of cognitive development in children, the last two of which are concrete operations and formal operations. Although children progress through these stages at different rates, the order of the stages is thought to be invariant. In the concrete operations stage, beginning roughly around 7 years of age, the child is capable of reasoning logically on classes and relations, based always on his perceived experience with objects under consideration. The concrete operational child also acquires conservation or invariance concepts of various quantitative physical properties, such as length, number, liquid and solid quantity, area, mass weight, and volume.

Beginning roughly around 11 years of age, the child begins to enter the formal operational stage (Piaget and Inhelder, 1969). Formal thought has four distinctive features: reality as a subset of possibility, hypothetico-deductive thought, propositional logic, and the combinatorial system.

4

Whereas a concrete operational child is oriented to concrete reality, the child in the formal operational stage is oriented to possiblity. For him, reality is a subset of all possible situations. Empirical results are interpreted in the light of all possible consequences. The formal thinker verifies reality by "casting possible situations in hypothetico-deductive statements (Sheehan, 1970)". He is capable of deductive reasoning on hypothetical statements, independent of his direct perception of their truth or meaning. He can apply logic to propositions, not just classes and relations.

An important feature of formal thought is the capability for combinatorial analysis. The formal operational child can generate the array of all possible combinations of relevant variables in a problem. The presence or absence of a combinatorial system in a subject's reasoning is cited by Inhelder and Piaget (1958, pp. 279-80) as the crucial difference between concrete operational and formal operational thinking. It is this combinatorial system which makes it possible for the child to consider variables systematically in a search for functional relationships between variables. The combinatorial operations make it possible for a subject to isolate each variable in turn while holding the other variables constant, and to do this systematically. In contrast, the concrete operational child faced with a multi-variable situation often fails to consider either completely or systematically the combinatorial array of possiblities. In an experiment conducted by Inhelder, where the children were to determine the variables affecting the rate of oscillation of a pendulum, "subjects at the level of concrete operations [varied] everything at the same time ...

(Piaget and Inhelder, 1969, p. 148)."

Mathematical Structure

A mathematical example of an array of possibilities is the operation table in certain mathematical systems. These systems consist of a set of elements together with a rule or function, called a binary operation, which assigns a unique result (one of the set of elements) to each pair (hence the name "binary") of inputs from the set of elements.

Although addition, subtraction, and other arithmetic operations are defined on an infinite set of numbers, binary operations defined on a finite set of elements are also of interest to mathematicians. Dienes and Jeeves (1965, 1970) developed a task with which they studied strategies for learning binary operations from certain finite mathematical systems. Their subjects learned the rules of the system through a series of moves, or trials, in a game-like situation. They set up a "machine" with a window where one of a small set of symbols was shown. This was the "state" of the machine. The subject (or learner) could choose one of the same set of symbols to "play". The "state" and "play" pair of symbols determined a unique result as the next "state". The subject was allowed to make successive plays, in an attempt to learn the "rules of the game", that is, to be able to predict the result for any given pair of state and play.

Strategy

Dienes and Jeeves described a hierarchy of three types of strategies, from lowest to highest: memory, where the subject learned each combination or pair separately, with no connections between the pairs; pattern, where the subject organized the set of pairs into subsets, each governed by its own rule or pattern; and operator, where the subject viewed each element as an "operator" transforming each of the symbols it was paired with, evidenced by the subject holding one of the symbols constant over several learning trails to investigate how it operated. In addition to the three "pure" strategies described above, mixed strategies were observed.

Mode of Presentation.

The subjects in Dienes and Jeeves' study were constrained in their choices by the result of the previous move, as noted by Branca and Kilpatrick (1972). Branca and Kilpatrick suggested that the strategy scores might have been somewhat insensitive to the actual strategy the subject wished to use. Kellogg (1973) and Jeeves (1971) both report that subjects allowed to freely choose both inputs learned significantly faster than in the state-play-state format. Kellogg reported consistency in strategy use across the two formats.

Either the subject or the experimenter can choose the input pairs in learning trials. When the subject has free choice of the input pairs (and their order) for all learning trials, the tasks is in a selection mode. If the experimenter selects the sequence of input pairs to be presented to the learner, the task is in a reception mode. This distinction is parallel to one made by Bruner (1956) in concept attainment tasks. He classified the task as reception or selection depending on whether the experimenter or the learner could choose and order the examples about which the learner was to receive information.

Since the learner controls the sequence of information which he examines and processes when using the selection mode, this mode is ideally useful in studying the learner's strategy. In particular, the formal operational child, with his capacity for combinatorial thought, should be able to consider the full range of possible input pairs, organizing and investigating them systematically. The concrete operational child is more likely to be satisfied with and only capable of an incomplete or unorganized examination in input pairs. The formal operational child should seek to isolate the effect of each of the two variables he controls (the two inputs) by holding one of them constant and varying the other. The strategy of holding one input constant while investigating the effect of varying the other input is what Dienes and Jeeves label an operator strategy.

PROBLEM

Since formal operational but not concrete operational children are expected to apply combinatorial operations in a multi-variable situation, an operator strategy should be used more often by the formal operational than the concrete operational children. The present study attempted to examine the validity of this conjecture. If there is a difference in strategies, it may be accompanied by increased efficiency in learning for formal operational children. This study examined whether, in a selection mode, formal operational children learn in fewer trials than concrete operational children.

Such a difference in learning efficiency may be due not only to the use of an operator strategy, but also to an increased capability to handle the information as it is structured by an operator strategy. By experimentally imposing an operator strategy, the reception mode can be used to study the learner's ability to process this structured information sequence. This study attempted to determine whether formal operational children use the information from an imposed operator strategy sequence more effectively, and thus learn in fewer trials than concrete operational children.

Within the limitations of its design, this study also attempted to provide information about the effect of presentation mode (selection vs. reception) and developmental stage (concrete vs. formal) on transfer to a later task.

PROCEDURES

The instrument used to classify students as concrete or formal operational was a minor modification of a test developed by Longeot (1965) and translated from French to English by Sheehan (1970). A few items were reworded for clarity and the four subparts of Sheehan's version were rearranged for ease of administration in the cooperating school. The test contained deductive reasoning items, proportional thinking items, and items requiring the listing of all logical possibilities for certain events (combinatorial items). Criteria supplied by Sheehan were used to classify students as concrete operational or formal operational.

Sample

The subjects were selected from the 305 eighth-grade mathematics students in a rural-suburban junior high school. Two hundred fifty-one completed all sections of the classification test. Using Sheehan's criteria (1970), 120 students were classified as concrete operational and 105 as formal operational, with 26 students unclassified.

A random sample of 20 students was drawn from the list of those pupils classified as concrete operational. Similarly, a random sample of 20 subjects was chosen from the formal operational list. Subjects were asked by the math team leader if they would be willing to participate. Three formal and six conrete subjects declined to participate and were replaced with randomly chosen replacements from the respective lists.

Experimental tasks

Each of the 40 subjects was presented individually with two experimental tasks, the second task approximately one week after the first. Both experimental tasks were presented using a gameboard with three columns. The gameboards for Task 1 and Task 2 are shown in Figure 1.

Each subject was told that he was to learn the rules of a game. Each time a picture in the first column and a picture in the second column were chosen (using markers on the gameboard), that pair would determine a certain answer or result in the third column, according to the rules of the game. A third marker in the final column was used for the subject's prediction and for the experimenter's feedback of the correct response to the given input pair.

Insert Figure 1

A learning trial consisted of three parts: (a) a choice of input pair (subject's or experimenter's choice, depending on the experimental treatment); (b) prediction of the answer by the subject; and (c) verification or correction of the answer by the experimenter. The particular rules or operations which the subjects were asked to learn are summarized by the operation tables in Figure 2. The mathematical systems illustrated are the cyclic groups of order 4 and 5, respectively.

Insert Figure 2 about here

Each task was presented in blocks of 32 learning trials, each block being followed by a criterion test to assess the subject's learning at that point. The subject continued receiving the 32-trial learning blocks until he passed a criterion test. The task was then terminated and the subject was interviewed.

Tasks were presented in either a Selection Mode, where the subject chose the input pairs and their order, or in a Reception Mode, where the input pairs and their order were pre-determined and presented by the experimenter. Each Selection subject was told he could choose to put the markers on any input pair and the correct answer would be indicated. He would receive answers for each pair he chose, in whatever order he thought would help him learn the rules of the game. Each Reception subject was told that the experimenter would show him the answer to the various pairs. He was to watch the results and attempt to learn the rules of the game.

Each criterion test was a set of trials selected from the operation table of the structure to be learned in the task. The experimenter presented an input pair and the subject predicted the answer. No corrections were made on individual pairs; the subjects was told at the end of the test how many he had done correctly, e.g., "6 out of 10". Not all possible pairs were tested on each criterion test, to lessen the amount of time spent on testing and thus make standard periodic tests reasonable. Preliminary study indicated that 10 trials for Task 1 and 15 trials for Task 2, if chosen to be representative of the mathematical structure, were sufficent to indicate reasonable mastery of the operation rule. The particular selection of trials was varied from one criterion test to the next.

After the learning of each task was completed, the subject was asked to describe the "rules of the game" as he saw them.

Design and Dependent Measures

The two main independent variables in this study were Piagetian stage of cognitive development and presentation mode of the first task. These are referred to as Stage and Mode, respectively, in subsequent discussions. The task assignments in this study can be pictured as shown in Figure 3.

Insert Figure 3 about here.

Dependent Measures

Many measures could be constructed from the trial-by-trial raw data available on each subject. This study was concerned with the measures described below.

Operator Score was designed to measure the learner's use of an operator strategy. Operator Score was the fractional part of all trials which were part of operator runs. In an operator run, one of the inputs was held constant throughout 3 or more consecutive trials. To obtain the Operator Score, the number of trials involved in such runs was divided by the total number of learning trials.

Trials to Criterion was designed to measure how many trials each subject needed to master the task. The actual number of trials completed was always a multiple of 32. Although subjects may have mastered the task in the middle of a block of trials, this was not revealed until the criterion test after the full block of 32 trials. Therefore, mastery of the task was operationally defined to occur at the earliest trial after which no more than one prediction error occurred. Trials to Criterion was defined to be the number of trials up to the point of mastery.

After each task, the subject was asked to describe any patterns or relationships he observed in the task. These Task Descriptions were classified by the experimenter according to categories which will be described in the results section.

Trials to Criterion

A Stage x Mode analysis of variance was conducted for Trials to Criterion on Task 1. Table 1 presents the results of this analysis, with corresponding cell means and standard deviations in Table 2. Both main effects, Stage and Mode, were significant ($p < .01$) while the interaction was negligible. Formal subjects learned in fewer trials than concrete subjects and Reception subjects learned in fewer trials than Selection subjects.

Insert Tables 1 and 2 about here.

Table 3 summarizes the results of a Stage x Mode analysis of variance for Trials to Criterion on Task 2. The corresponding descriptive statistics are presented in Table 4. A significant Stage effect ($p < .05$) for Trials to Criterion favored the formal subjects. Reception subjects learned in fewer trials than Selection subjects but the difference was not significant ($.15 < p < .20$). The Stage x Mode interaction effect approaches significance ($p < .11$). A simple effects analysis (Winer, 1971, pp. 436-441) indicated that the concrete-formal difference was significant only for Selection subjects and that the Selection-Reception difference was significant only for concrete subjects.

Insert Tables 3 and 4 about here

Operator Score

Operator Score was obtained only for subjects with free choice of learning trials, i.e., subjects in the Selection Mode. Thus, no Mode factor was present for Operator Score on Task 1. The results of a one-factor (Stage) analysis of variance of Operator Score on Task 1 are presented in Table 5. Formal subjects had a higher mean Operator Score than concrete subjects (.52 versus .36) but the difference was not significant ($.15 < p < .20$).

Insert Table 5 about here

Since Task 2 as presented completely in Selection Mode, Operator Score was available for all 40 subjects. Table 6 summarizes the Stage x Mode analysis of variance for Operator Score on Task 2; Table 7 presents the corresponding cell means and standard deviations. Formal subjects had a significantly higher mean Task 2 Operator Score ($p < .05$) than concrete subjects. Both the Mode and Stage x Mode effects were negligible for Task 2 Operator Score.

Insert Tables 6 and 7 about here

Student Descriptions of the Task

After Task 1 and again after Task 2, each subject was asked to describe the "rules of the game" as he saw them. He was encouraged to tell everything he noticed about how the game worked. These descriptions were classified by the experimenter into the following categories:

Operator. The subject described each symbol from one column in terms of how it operated on the symbols in the other column. For example, several subjects said that the symbol '?' moved everything one space up on the board.

Counting. The subject obtained each result by counting squares on the playing board up or down the columns from one or more standard reference pairs or from the previous pair. Subjects often used the doubles such as (\$,\$) and (☽, ☽) as reference points from which to count.

Pattern. The subject formed sets of related pairs, using a known result for one pair to help find the result to another. Some of the relationships used were commutativity, symmetry, and the identity property. To be classified as a Pattern description, there should be no isolated pairs left unrelated to another pair or pairs.

Pattern-Memory. Some subjects used a pattern description for only part of the task pairs and individually memorized the others. Often the only pattern described was the identity property.

Memory. The subject memorized each pair separately, indicating no awareness of any relationships.

For the purposes of the present analysis, a simpler three level system of classification was used: Operator, Counting-Pattern, and Pattern-Memory (including full Memory descriptions). The numbers of subjects in each Stage x Mode cell of the experiment who gave descriptions in each of the three categories are presented in Table 8.

Insert Table 8 about here

To determine whether the task descriptions were independent of Stage and Mode the χ^2 statistic was applied to the appropriate contingency tables, with the results presented in Table 9.

Insert Table 9 about here

On both tasks, formal subjects gave more Operator descriptions and fewer Pattern-Memory descriptions than the concrete subjects. On Task 1, Reception subjects gave more Operator descriptions and fewer Pattern-Memory descriptions than the Selection subjects. On Task 2, Reception subjects tended to give more Operator and fewer Counting-Pattern descriptions than Selection subjects.

In order to examine the relationship between descriptions on Task 1 and descriptions on Task 2, a contingency table (Table 10) of Task 1 by Task 2 descriptions was constructed.

Insert Table 10 about here

Twenty-eight of the forty subjects gave descriptions in the same category on both tasks. If the categories are viewed as forming a hierarchical scale with Operator as the highest level and Pattern-Memory as the lowest, only three subjects changed to a lower level on Task 2, while nine changed to changed to a higher level description. Nineteen of the twenty-one subjects who gave high level (Operator or Counting-Pattern) descriptions on

Task 1 also gave high level descriptions on Task 2. Only ten of the nineteen subjects who gave low level (Pattern-Memory) descriptions on Task 1 also gave a low level description on Task 2.

DISCUSSION

A primary purpose of this study was to determine whether there were differences in strategy use between concrete and formal operational children. The evidence supports the hypothesized strategy differences, with formal subjects having a higher mean Operator Score than concrete subjects on both tasks. The overall difference was not significant ($.15 < p < .20$) on Task 1, while it was on Task 2 ($p < .05$). It should be noted that this comparison involved only 20 subjects on Task 1 but 40 subjects on Task 2.

A more complex task with perhaps more variables may have shown a stronger strategy difference. Inhelder and Piaget (1958) suggest that children in the late concrete stage might be able to systematically explore one or possibly two variables, but probably not as many as three to five. They note (p. 63) that multiplication of concrete relations may be used when two variables, such as length and thickness, are homogeneous (that is, both are spatial) and a lesser amount of one can be compensated additively by a gain in the other. Formal thought is required to handle compensations between heterogeneous factors such as length and density.

The two variables (first and second input) in each task of the present study are certainly homogeneous and their action does compensate additively for each other: moving up one symbol in the first column is compensated for by moving down one in the second column. Thus, the particular format used for the tasks in this study may have rendered them more easily solvable by advanced concrete operational thinking.

Despite the fact that Reception subjects learned more quickly and were exposed to an operator strategy sequence on Task 1, there was no difference between the effect of Reception and Selection training on the use of an operator strategy on the second (transfer) task. It is not known whether the experience in Task 1 had any actual effect on operator strategy use on the second task, since no group received the transfer task without previous training. The fact that Reception-trained subjects had Operator scores as high as the Selection-trained subjects may be due to the particular sequence used in the Reception Mode in this study. Certainly other sequences may have different effects on both initial learning and transfer to later tasks.

It was suggested in a previous section that formal subjects could apply combinatorial thought to systematically explore a Selection Mode task and would thus learn it more quickly than concrete subjects. The evidence from Task 1 and Task 2 indicates that formal subjects did learn in fewer trials than concrete subjects in Selection Mode. The results also indicate that formal subjects were able to take advantage of the structured data in the Reception Mode and learn in fewer trials than concrete subjects in the Reception Mode. Generally, formal operational children not only adopted an operator strategy more than concrete operational children, they also used the resulting information more effectively.

The advantage of an operator strategy for learning a task lies in the organization it imposes on the data, enabling the subject to identify the role of each of the variables. Even if concrete subjects do not spontaneously adopt operator strategies, they may be capable of using the information as it is structured by an imposed operator strategy. In this study, concrete subjects learned more quickly in the Reception Mode than in the Selection Mode. For these concrete operational children, the imposed operator strategy sequence, by holding one variable constant, may have reduced the complexity of the problem so that it could be handled more easily by concrete operational thought.

Formal operational subjects also learned more effectively in the Reception Mode than in the Selection Mode. In the present study formal subjects may have become preoccupied with the planning of a strategy in Selection Mode or they may have experienced difficulty in coordinating the search aspect and the information processing aspect of the task. Furthermore, formal thought is probably not fully developed and stable for eighth-grade children. They may not be able to apply formal thought readily on a novel task and may be inefficient in using it. Tagatz (1967) found that, for fifth and sixth grade subjects trained in a concept attainment strategy requiring formal operational logic, those who used this strategy took more trials to learn than those who reverted to a strategy dependent only on concrete operational logic.

The results for Trials to Criterion on Task 2 present a different pattern than that for Task 1. Among formal subjects, those who had received Selection training learned Task 2 in slightly fewer trials than those who had received Reception training. On the other hand, among concrete subjects, those with Reception training learned considerably faster than those with Selection training. For Task 1, the superiority of Reception Mode subjects on Trials to Criterion was explained by the application of concrete logic to the structured data from an imposed operator strategy. However, Task 2 was in the Selection Mode for all subjects, so concrete logic was not longer sufficient. Reception training did not induce a higher level of operator strategy use on Task 2. The mechanism by which concrete subjects benefited from the Reception Mode seems to be by means of the transferable insights or generalizations which these subjects were able to achieve in the Reception Mode on Task 1 more easily than in the Selection Mode. Reception-trained subjects gave more high level (Operator, Counting, or Pattern) descriptions for Task 1 then Selection-trained subjects, especially among concrete subjects. Subjects giving these high level descriptions for Task 1 had generalizations available which could be applied to Task 2. Subjects giving low level (Pattern-Memory or Memory) descriptions for Task 1 had either no generalizations or one of limited scope to transfer to Task 2. The advantage of the high level Task 1 description for the learning of Task 2 can be seen in Table 11. This table presents the mean trials to Criterion and mean Operator Score for Task 2 for those subjects who gave each of the three levels of Task 1 descriptions.

Insert Table 11 about here

Analysis of variance and Newman-Keuls post hoc analyses indicated that groups 1 (Operator) and 2 (Counting-Pattern) learned significantly faster ($p < .05$) than group 3 (Pattern-Memory) but did not differ from each other. There were no significant differences in Operator Score among the three groups. Thus, the high level descriptions obtained on Task 1 seem to be beneficial for learning efficienty on Task 2, but not through the greater use of an operator strategy.

The effect of the transfer of task generalizations can be seen very clearly in the Concrete-Reception cell. The five Concrete-Reception subjects who gave a high level Task 1 description had a mean Trials to Criterion of 44.4 on Task 2, while the other five Concrete-Reception subjects had a mean Trials to Criterion of 185.4 on Task 2. For comparison, the 10 Concrete-Selection subjects, of which only one gave a high level Task 1 description, had a mean Trials to Criterion of 185.5 for Task 2. Thus, the advantage of Reception training for concrete subjects for Task 2 Trials to Criterion can be attributed completely to the performance of the subjects who gave high level Task 1 descriptions.

While formal Reception Mode subjects learned faster than formal Selection Mode subjects on Task 1, their performance on Task 2 was essentially equivalent. Selection-trained formal subjects may have been less distracted by the search aspect on Task 2 because of their experience with Task 1 in the Selection Mode. At the same time, Reception-trained subjects had to adapt to the more demanding Selection Mode. It seems very possible, however, that the principal reason for a lack of difference in transfer effect between the two levels of Mode among formal subjects on Task 2 was that their scores approached a realistic minimum number of trials required to master Task 2. Mastery of Task 2 within the first two blocks (64 trials) required immediate solution of the task with little experimentation. This was accomplished only by subjects who had achieved very firm and transferable generalizations which they remembered and were able to apply approximately a week after they completed Task 1. Students who had not achieved such generalizations on Task 1 required longer to learn, as indicated earlier in the discussion of Table 11.

In summary, Reception training with an imposed operator strategy seems to lead to more efficient initial learning, but not to stimulate a greater use of an operator strategy on a transfer task than does Selection training. Reception training seems to lead to more efficient learning on a transfer task only if the learner has achieved generalitzations which are transferable from training to transfer task. In this study, the two tasks had similar solutions. Use of less similar task might be expected to yield quite different transfer results for both learning efficiency and strategy use. Also, Reception training using other types of learning trial sequences may not result in similar transfer effects.

In connection with the overall male-female differences found on Task 1, the initial difficulty on the part of female subjects may have been due to lack of experience with task requiring active experimentation or to the inhibiting effect of the presence of a male adult (the experimenter) on the females during the task. In either case, the experience of Task 1 seems to have been sufficient to eliminate the differences.

Conclusions

The principal conclusions of this study are summarized below.

1. Formal subjects learned in fewer trials and tended to use an

operator strategy more than concrete subjects.

2. Subjects in the Reception Mode learned more quickly than subjects in the Selection Mode, while there was no difference between Reception and Selection training in stimulating use of an operator strategy on a subsequent task.

3. Reception training using an imposed operator strategy guided subjects to more transferable generalizations than Selection training, resulting in more efficient learning on the transfer task among concrete subjects.

Generalizations of these conclusions must be qualified by the choice of tasks and sample size used in this study. A different pattern of transfer effects might be found for less structurally similar tasks, since the performance of subjects on Task 2 seems to have been related to the type of insight gained on Task 1. Also, despite the small sample size necessitated by time-consuming interviews, the study has laid a basis for further research.

Only one type of sequence was used for the Reception training in this study. The effect of other types of Reception mode sequences and methods of training on both learning efficiency and strategy use needs to be investigated. Typical classroom approaches such as exposition and demonstration as well as the use of concrete models might be examined and compared. Emphasis should be placed on the ability of subjects to transfer strategies to types of problems besides those on which they are trained.

Comparisons between formal and concrete thinkers should be extended to other age levels, especially older adolescents and adults. The type of research design used in this study, where subjects within a limited age range were classified as concrete or formal operational and then compared, should be used more widely to study strategy differences as well as the ability to learn or discover mathematical concepts. Differences obtained simply by comparing subjects at different age levels can be considered only suggestive of changes actually due to cognitive development, which is confounded with other age- and experience-related factors.

The relationship between cognitive development and the thought processes used in mathematical learning may be studied by embedding mathematical concepts in tasks that reveal choices and decisions mirroring cognitive processes as well as cognitive products. The study of strategy sequences in this experiment was an attempt at this approach.

Finally, the fine structure of Piaget's construct of formal operations needs further research and explication. This will hopefully lead to more reliable and valid group assessment procedures, yielding more detailed developmental information for both research and applications.

BIBLIOGRAPHY

Branca, N., and Kilpatrick, J. The consistency of strategies in the learning of mathematical structures. Journal for Research in Mathematics Education, 1972, 3, 132-140.

Bruner, J. S., Goodnow, J. J., and Austin, G. A. A Study of Thinking. New York: Wiley, 1956.

Chilewski, R. J. Consistency of strategies across mathematical structures and embodiments. (Doctoral dissertation, Northwestern University) Ann

Arbor, Mich.: University Microfilms, 1973, No. 74-7724.

Chrisman, Gerry L. The Effect of Concrete and Formal Operational Thought on Learning Strategies in Mathematical Structures. (Doctoral dissertation, Purdue University) Ann Arbor, Michigan: University Microfilms, 1975, No. 75-10859.

Dienes, Z. P. and Jeeves, M. A. Thinking in structures. London: Hutchinson Educational, 1965.

Dienes, Z. P. and Jeeves, M. A. The effects of structural relations on transfer. London: Hutchinson Educational, 1970.

Flavell, J. The Developmental psychology of Jean Piaget. New York: Van Nostrand, 1963.

Inhelder, B. and Piaget, J. The growth of logical thinking from childhood to adolescence. New York: Basic Books, 1958.

Inhelder, B. and Piaget, J. The early growth of logic in the child. New York: Norton, 1964.

Jeeves, M. A. Some experiments on structured learning. Educational Studies in Mathematics, 1971, 3, 454-476.

Kellogg, H. M. Task format and the deep-end hypothesis in the learning of mathematical group structures. (Doctoral dissertaion, Columbia University) Ann Arbor, Mich.: University Microfilms, 1973, No. 74-6403.

Leskow, S. and Smock, C. D. Development changes in problem-solving strategies: Permutation. Developmental Psychology, 1970, 2, 412-422.

Longeot, F. Analyse statistique de trois tests genetiques collectifs. Bulletin de L'Institu National de'Etude du Travail et D'Orientation Professionelle, 1964, 20, 219-232.

Lovell, K. R. Intellectural growth and understanding mathematics. Journal for Research in Mathematics Education, 1972, 3, 164-182.

Piaget, J. and Inhelder, B. The psychology of the child. New York: Basic Books, 1969.

Sheehan, D. J. The effectiveness of concrete and formal instruction procedures with concrete-and formal-operational students. (Doctoral dissertation, State University of New York at Albany) Ann Arbor, Mich.: University Microfilms, 1970, No. 70-25479.

Tagatz, G. E. Effects of strategy, sex, and age on conceptual behavior of elementary school children. Journal of Educational Psychology, 1967, 58, 103-109.

Winer, B. J. Statistical principles in experimental design. (2nd ed.) New York: McGraw-Hill, 1971.

Yudin, L. W. Formal thought in adolescence as a function of intelligence. Child Development, 1966, 37, 697-708.

Yudin, L. W. and Kates, S. Concept attainment and adolescent development. Journal of Educational Psychology, 1963, 54, 177-182.

TABLE 1

2 (Stage) × 2 (Mode) ANOVA for Task 1 Trials to Criterion

Source	df	MS	F-ratio
Stage	1	27667.6	7.96***
Mode	1	26522.5	7.63***
Stage x Mode	1	532.9	.15
Within (Error)	36	3475.2	
Total	39		

*** $p < .01$

TABLE 2

Task 1 Trials to Criterion Means and Standard Deviations*

(standard deviations in parentheses)

	Selection	Reception	Total
Formal	157.5 (65.9)	98.7 (45.6)	128.1 (62.8)
Concrete	202.8 (53.6)	158.6 (68.0)	180.7 (63.7)
Total	180.2 (62.9)	128.7 (64.1)	

* 10 subjects per cell

TABLE 3

2 (Stage) x 2 (Mode) ANOVA for Task 2 Trials to Criterion

Source	df	MS	F-ratio
Stage	1	35046.4	6.48**
Mode	1	10758.4	1.99
Stage x Mode	1	14288.4	2.64
Within (Error)	36	5411.8	
Total	39		

** $p < .05$

TABLE 4

Task 2 Trials to Criterion Means and Standard Deviations*
(standard deviations in parentheses)

	Selection	Reception	Total
Formal	88.5 (48.9)	93.5 (71.6)	91.0 (59.7)
Concrete	185.5 (79.2)	114.9 (88.7)	150.2 (89.5)
Total	137.0 (81.1)	104.2 (79.2)	

* 10 subjects per cell

TABLE 5

ANOVA for Task 1 Operator Score

Source	df	MS	F-ratio
Stage	1	.115	2.10
Within (Error)	18	.055	
Total	19		

TABLE 6

2 (Stage) x 2 (Mode) ANOVA for Task 2 Operator Score

Source	df	MS	F-ratio
Stage	1	.409	6.05**
Mode	1	.024	.35
Stage x Mode	1	.000	.00
Within (Error)	36	.068	
Total	39		

** $p < .05$

TABLE 7

Task 2 Operator Score Means and Standard Deviations*
(standard deviations in parentheses)

	Selection	Reception	Total
Formal	.56 (.22)	.61 (.25)	.59 (.23)
Concrete	.36 (.23)	.41 (.33)	.38 (.28)
Total	.46 (.24)	.51 (.30)	

* 10 subjects per cell

TABLE 8

Numbers of Students Giving Each Type of Description on Task 1 and Task 2

		Operator	Counting-Pattern	Pattern-Memory
Task 1				
Formal	Selection	3	3	4
	Reception	7	2	1
Concrete	Selection	0	1	9
	Reception	4	1	5
Task 2				
Formal	Selection	3	7	0
	Reception	6	2	2
Concrete	Selection	0	3	7
	Reception	4	3	3

TABLE 9

Contingency Table for Description by Stage and Description by Mode

		Operator	Counting-Pattern	Pattern-Memory	χ^2_{obs}
Task 1					
	Formal	10	5	5	8.12**
	Concrete	4	2	14	
	Selection	3	4	13	7.29**
	Reception	11	3	6	
Task 2					
	Formal	9	9	2	7.86**
	Concrete	4	6	10	
	Selection	3	10	7	5.77*
	Reception	10	5	5	

* $p < .10$, since for df = 2, $\chi^2_{.90} = 4.6$

** $p < .05$, since for df = 2, $\chi^2_{.95} = 6.0$

TABLE 10

Contingency Table for Student Descriptions, Task 1 by Task 2

Task 1 Descriptions	Task 2 Descriptions: Operator	Counting-Pattern	Pattern-Memory
Operator	12	1	1
Counting-Pattern	0	6	1
Pattern-Memory	1	8	10

TABLE 11

Mean Trials to Criterion and Operator Score on Task 2 for Groups Based on Task 1 Descriptions

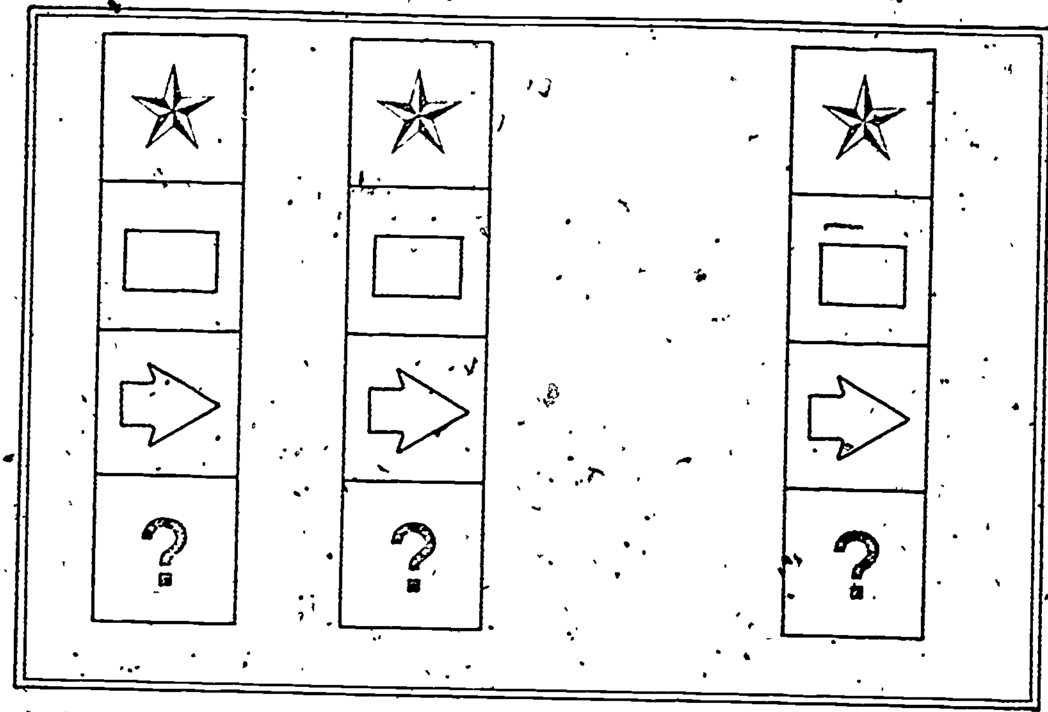
Group	Task 1 Description	Trials to Criterion Task 2	Operator Score Task 2
1	Operator	69.7	.47
2	Counting-Pattern	98.9	.42
3	Pattern-Memory	166.1	.57

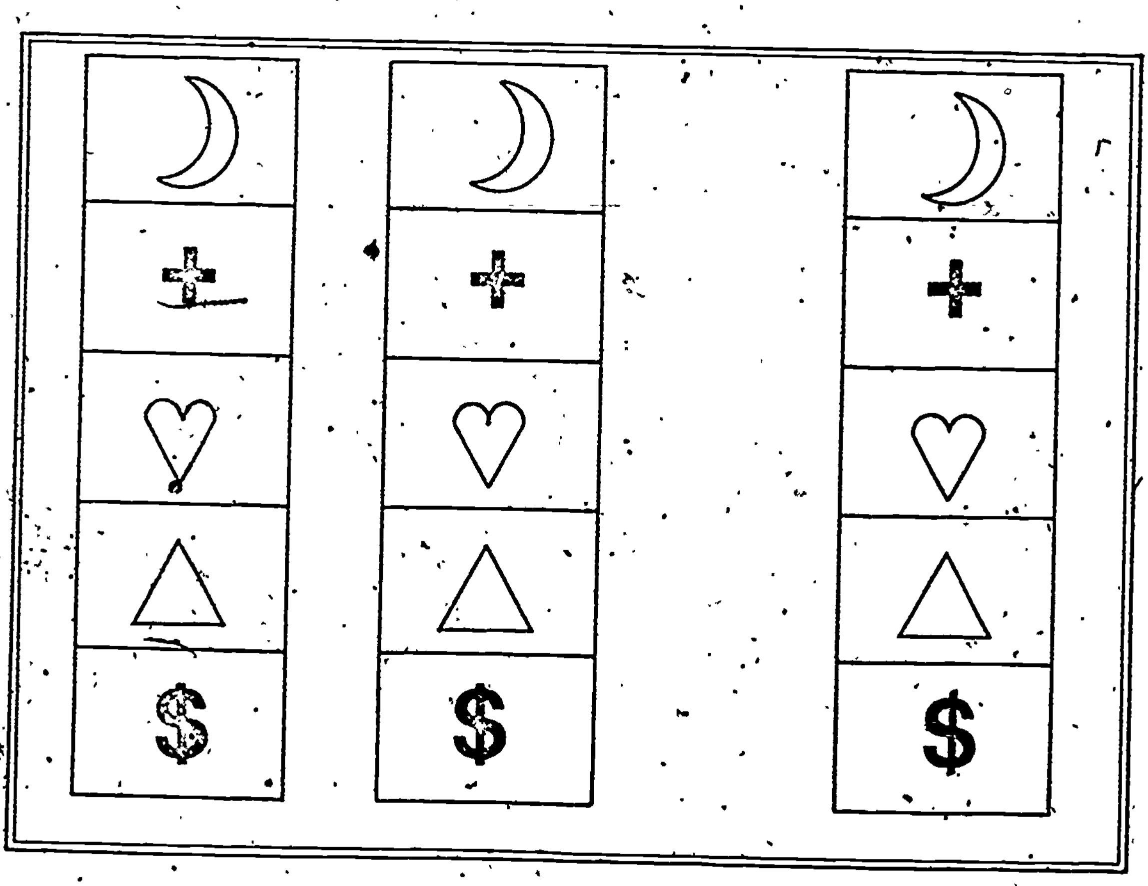
Figure Captions

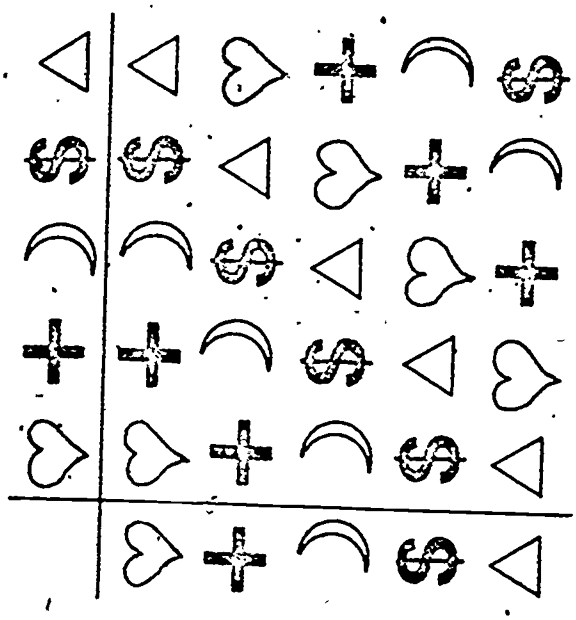
Fig. 1. The Gameboards for Task 1 (top) and Task 2 (bottom)

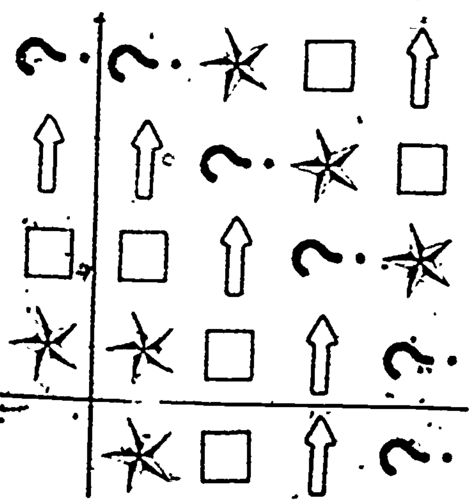
Fig. 2. Operation Tables for Task 1 and Task 2

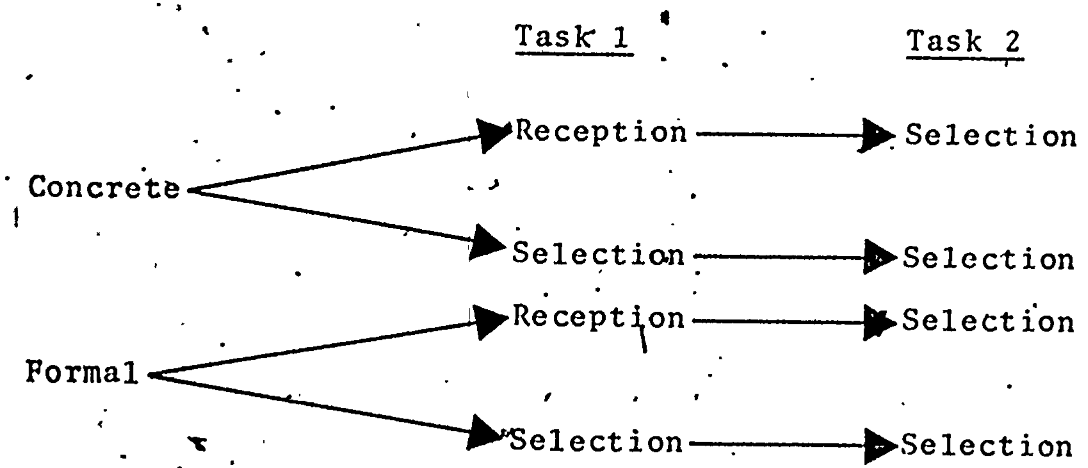
Fig. 3. The Task Assignments










Task 1
Task 2
Concrete
Reception
Selection
Selection
Selection
Formal
Reception
Selection
Selection
Selection